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An Efficient Structural Resizing Procedure for Meeting Static Aeroelastic Design Objectives

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As part of a continuing effort to develop automated finite-element resizing procedures for improved aeroelastic behavior, techniques for addressing control-surface effectiveness, flexible-surface lift-curve slope, and static divergence velocity have been developed and demonstrated on a preliminary-design representation of a wing having advanced-composite cover skins. Starting with a design that has been sized for strength requirements only, the techniques assign additional material to the most effective composite fiber directions at the most effective locations in the structure. The overall procedure, which is an outgrowth of optimization technology developed for flutter resizing in the Flutter And Strength Optimization Program (FASTOP), exhibits excellent convergence characteristics, in that a specified design requirement is satisfied with a minimum weight increase in only a few resizing cycles.

Introduction

DURING the development of the FASTOP computer program system for the Air Force Flight Dynamics Laboratory,[1-3] a simple and highly effective structural resizing method was developed for increasing the flutter speed of a strength-based design of a lifting surface to a desired value with a minimum increase in weight. Because of the generality of the method, it was natural to extend its application to other aeroelastic design problems. With the increasing use of advanced composite materials in military aircraft, this work became particularly important, since these materials introduce the potential for aeroelastic deficiencies not normally present in metallic designs. The work was further motivated by Air Force interest[4,5] and NASA interest (in its Highly Maneuverable Aircraft Technology, HiMAT, project) in exploiting the directional properties of composites actually to produce favorable aeroelastic characteristics not attainable with metals. An interesting example is the optimum use of composites to avoid the classic divergence problem encountered in a forward-swept wing.

Accordingly, work was undertaken at Grumman to develop structural finite-element resizing techniques that address any one of the following static aeroelastic design objectives: satisfying a generalized displacement requirement (e.g., a prescribed twist at a given wing station to produce a desired lift distribution), improving control-surface effectiveness, improving flexible-surface lift-curve slope or, increasing divergence velocity. For the first of these objectives, a fully automated procedure was developed and integrated into the latest version of the Automated Structural Optimization Program (ASOP-3).[6,7]

For the remaining three objectives, a general resizing approach was developed and incorporated into a new pilot computer program. Starting with a strength-adequate design, the program determines where and to what extent material should be added to satisfy a specified static aeroelastic constraint with a minimum increase in weight. While composites are not treated explicitly, these materials are handled by grouping laminae having a common fiber direction into subelements of each composite element in the structure.

This paper describes the general resizing approach used, and provides specific details concerning each of the last three listed objectives. Examples are given which demonstrate the rapid convergence characteristics of the method and show its potential as a practical design tool.

General Resizing Approach

The general resizing approach to be presented is based on a simple optimality criterion. It has been shown[1,8] that, for the case of a single behavioral constraint (e.g., a critical flutter speed or a particular structural deflection), and in the absence of other constraints, a minimum-weight design is achieved when the partial derivative of the behavior function with respect to element weight has the same value for all elements. That is,

$$\frac{\partial F}{\partial w_i} = C \qquad (i=1,2,...,n) \tag{1}$$

where F is the behavior function being constrained, w_i is the weight of the ith finite element (of a total of n elements), and C is a constant.

When, as in most practical designs, there are strength constraints and minimum and/or maximum gage constraints in addition to the behavioral constraint, the uniform-derivative criterion expressed in Eq. (1) can be applied to the set of elements not governed by these other constraints, the corresponding element weights being referred to as the active variables. In that case, the criterion is less rigorously applicable, but should still give a design of nearly minimum weight.

The general resizing formula used here for achieving the desired state of uniform derivatives for a particular aeroelastic constraint follows from a physical interpretation of the terms appearing in the associated derivative expression, and the approximate manner in which the derivatives change with changes in element weight. This is analogous to the underlying logic of the basic stress-ratio resizing formula used to obtain a fully stressed design, viz:

$$w_{i_{\text{new}}} = \left(\frac{\sigma_{i_{\text{old}}}}{\sigma_{i_{\text{allowable}}}} \right) w_{i_{\text{old}}} \tag{2}$$

where $w_{i_{\text{old}}}$ and $w_{i_{\text{new}}}$ are the weights of the ith element before and after a resizing step, respectively; $\sigma_{i_{\text{old}}}$ is the representative element stress before resizing; and $\sigma_{i_{\text{allowable}}}$ is the

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desired value of the representative element stress after resizing.

Equation (2) is based on the assumption that the internal forces in the finite elements remain unchanged during resizing, an assumption that is valid only when the structure is statically determinate. A single application of this formula to an arbitrary starting design will never yield a fully stressed design in a statically indeterminante structure. However, its use in an iterative way has generally shown it to exhibit excellent convergence characteristics, leading to a nearly fully stressed design in just a few resizing cycles. In the approach presented now for meeting static aeroelastic design objectives with near-minimum weight, there are similarities to the fully stressed design approach for near-optimum strength design.

Starting with a design based on strength and minimum manufacturing gage constraints only, subsequent resizing of a structure to satisfy stiffness-related requirements makes use of the following formula, which is applied in several cycles:

$$w_{i_{\text{new}}} = \left(\frac{(\partial F/\partial w_i)_{\text{old}}}{(\partial F/\partial w)_{\text{target}}} \right)^p w_{i_{\text{old}}} \tag{3}$$

Here, $w_{i_{\text{old}}}$ and $w_{i_{\text{new}}}$ are the weights of the ith finite element before and after resizing in the current cycle; $(\partial F/\partial w_i)_{\text{old}}$ is the partial derivative of the aeroelastic behavior function (e.g., control-surface effectiveness, divergence speed, etc.) with respect to w_i, computed for the design existing prior to resizing in the current cycle; and $(\partial F/\partial w)_{\text{target}}$ is a quantity referred to as the "target derivative," which is the same for all elements and which is discussed below. The exponent "p" is determined from an approximation of how the derivative for an element varies with its weight. When the constraint is on control-surface effectiveness or flexible-surface lift-curve slope, $p=1$; for the constraint on divergence velocity, $p=\frac{1}{2}$. More will be said about this exponent later.

At the optimum design, the target derivative $(\partial F/\partial w)_{\text{target}}$ will be the constant C in Eq. (1), which is the common value of the derivatives $\partial F/\partial w_i$ for all active design variables. However, prior to convergence to an optimum design, the element derivatives will differ from each other in value and perhaps in sign as well. As the target derivative is not known until the optimum design is achieved, it is necessary to resort to an iterative procedure to find a value for it. This procedure starts by using an assumed value for the target derivative and applying Eq. (3) to obtain a tentative new design. The incremental change in the aeroelastic behavior parameter associated with this tentative design is then approximated by the linear relationship

$$\delta F = \sum_{i=1}^{n} \left(\frac{\partial F}{\partial w_i} \right) (w_{i_{new}} - w_{i_{\text{old}}}) \tag{4}$$

If this predicted increment is within a specified tolerance of the desired increment, the tentative new design is accepted. Otherwise, the target derivative is adjusted automatically and the process is repeated.

Equation (4) provides a rapidly computed first-order approximation to the change in F. Alternatively, an exact value could have been obtained by performing the required aeroelastic analysis for the tentative new design. However, as this computationally expensive operation would have to be performed for each trial value of the target derivative, it is highly advantageous and, in practice, satisfactory to use Eq. (4) instead. Only after a tentative design has been accepted is an exact value for F computed.

Generally, the procedure just outlined is used to increase algebraically the value of an aeroelastic behavior parameter by selectively adding material to an initial strength-based design. The procedure is applied in several cycles, with some elements possibly having material first added and later removed, but no element gage is ever permitted to be reduced below its value in the initial design. In the final design, when the behavioral constraint has been satisfied and the desired state of uniform derivatives has been nearly achieved for all resized elements, it is logical to expect the derivatives for those elements to be positive in sign, since adding more material to them should increase further the magnitude of the aeroelastic behavior parameter. Thus, the target derivative in Eq. (3) will be a positive quantity. This presents no special problem for elements having negative derivatives, as might first appear when one examines the equation. For such elements, it is obvious that their gages will be governed by strength or minimum-gage requirements and they will thus not be subject to resizing by Eq. (3).

The desired change in the behavior function in a single resizing step need not be the total change required. It has been found advantageous to move from the initial value of the behavior function to the vicinity of the constraint value in a series of small steps, rather than a single large step. If this is done, Eq. (4) provides a better approximation in each step. Furthermore, when the neighborhood of the constraint boundary is first reached, the design is usually close to the minimum-weight design. Accordingly, the procedure employs a step-size option whereby the user specifies the number of approximately equal resizing steps to be performed in reaching the neighborhood of the constraint boundary. Beyond that point, an additional number of resizing steps are taken to enfoce convergence to a minimum-weight design. Convergence is considered to be achieved when the reduction in weight between two successive cycles becomes tolerably small or when a user-specified maximum number of resizing steps have been taken.

When composite elements are included in the structures model, each element is treated as a stack of separate elements attached at the same node points. Each element in a stack represents a group of plies having a common fiber direction, and its weight is treated as an independent design variable in Eqs. (3) and (4).

Development of Specific Relationships

The relationships needed to apply the general resizing procedure to meeting specific static aeroelastic design objectives are now presented.

Flexible-Surface Aerodynamic Influence Coefficients

Consider a flexible lifting surface modeled aerodynamically as a grid of lifting elements, or panels. Let $\{\alpha\}$ represent a vector of imposed angles of attack for the panels, measured from an equilibrium position. We now seek a direct matrix relationship that defines the resulting pressure distribution over the flexible surface. This relationship may be cast in the form

$$\{p\} = q[AIC]_F\{\alpha\} \tag{5}$$

where $\{p\}$ is the vector of resulting pressures on the panels, q is dynamic pressure $(\rho V^2/2)$, and $[AIC]_F$ is the flexible-surface aerodynamic influence coefficient matrix to be determined.

Using an appropriate linear aerodynamic theory, a rigid-surface aerodynamic influence coefficient matrix $[AIC]_R$ may be determined in a routine manner. Using this matrix, the flexible-surface pressure distribution may be expressed as the sum of the pressure due to the imposed angles of attack $\{\alpha\}$, as if the surface were rigid, and the pressure due to deformations $\{\Delta\alpha\}$. Thus,

$$\{p\} = q[AIC]_R\{\alpha\} + q[AIC]_R\{\Delta\alpha\} \tag{6}$$

But the angles of attack due to surface deformation are expressible in terms of the pressure distribution via a structural influence coefficient matrix $[SIC]$, whose columns give panel angles of attack due to unit panel forces. That is,

$$\{\Delta\alpha\} = [SIC] \ulcorner a \lrcorner \{p\} \tag{7}$$

where $\ulcorner a \lrcorner$ is a diagonal matrix of panel areas. Solving Eqs. (6) and (7) for $\{p\}$, the required flexible-surface aerodynamic influence coefficient matrix of Eq. (5) may be expressed in terms of known quantities:

$$[AIC]_F = \left[[I] - q[AIC]_R [SIC] \ulcorner a \lrcorner \right]^{-1} [AIC]_R \tag{8}$$

where $[I]$ is the identity matrix.

Since the structures and aerodynamics models of a lifting surface are not generally the same, it is necessary to define transformations between the two models for the purpose of computing the flexibility matrix $[SIC]$. At Grumman, these transformations are based on assumed relationships that transfer forces and moments at aerodynamic panel points to statically equivalent node forces in the structures model. Defining $[T_F]$ as a matrix in which a column contains structures model node forces due to a unit force acting normal to a panel in the aerodynamics model, and $[T_M]$ as a matrix in which a column contains similar node forces due to a unit streamwise moment acting on a panel, it can be shown that the following expression gives the desired flexibility matrix in terms of the structures model stiffness matrix $[K]$:

$$[SIC] = [T_M]^T [K]^{-1} [T_F] \tag{9}$$

Control-Surface Effectiveness

For an imposed unit deflection of a control surface represented by a group of panels in the aerodynamics model of a lifting surface, the rigid lift (or moment) L_R acting on the complete surface is given by

$$L_R = q\{a'\}^T [AIC]_R \{\alpha\} \tag{10}$$

The coefficients in $\{a'\}$ are panel areas, or products of panel areas and moment arms if the control-surface parameter is a moment, as in the case of an aileron. The imposed angle-of-attack vector $\{\alpha\}$ contains unit values for panels in the control surface and zeros for all others. An expression similar to Eq. (10) can be written for the flexible-surface lift by replacing $[AIC]_R$ with $[AIC]_F$ as given in Eq. (8). The control-surface effectiveness η (i.e., the ratio of flexible to rigid lift or moment) may then be expressed as

$$\eta = (q/L_R)\{a'\}^T [AIC]_F \{\alpha\} \tag{11}$$

If Eq. (11) is differentiated with respect to the weights of the structural finite elements, and use is made of Eqs. (8) and (9), the resulting control-surface effectiveness derivatives may be cast in the following form:

$$\frac{\partial \eta}{\partial w_i} = -\frac{1}{L_R}\{Q\}^T \frac{\partial [K]}{\partial w_i}\{\Delta\} \tag{12}$$

where

$$\{Q\}^T = q\{a'\}^T [AIC]_F [T_M]^T [K]^{-1} \tag{13}$$

and

$$\{\Delta\} = q[K]^{-1} [T_F] \ulcorner a \lrcorner [AIC]_F \{\alpha\} \tag{14}$$

Furthermore, if the elements of the stiffness matrix $[K]$ are linear functions of the design variables w_i, as they usually are, and $[k_i]$ is the stiffness matrix of the ith element, $\partial[K]/\partial w_i$ is equal to $[k_i]/w_i$. Thus, the derivative expression of Eq. (12) may be written in the form

$$\frac{\partial \eta}{\partial w_i} = -\frac{1}{w_i L_R}\{Q_i\}^T [k_i] \{\Delta_i\} \tag{15}$$

where the subscript in the Q and Δ vectors indicates that the only components included are those associated with the degrees of freedom of the ith element.

In Eq. (15) terms have been grouped in such a way as to aid in a physical interpretation of the quantities that appear, the ultimate intent being to estimate how the derivatives vary with changes in element weights. Examination of Eq. (13) reveals that the elements of $\{Q\}^T$ are values of total lift (or moment) acting on the complete surface due to unit loads acting at structures model degrees of freedom. Intuitively, it seems very unlikely that the resizing of an element will have any significant effect on these values of total lift or moment. It then follows that no significant changes in the derivative of that element will result from changes in $\{Q_i\}^T$.

Next, an examination of Eq. (14) reveals that the quantities in $\{\Delta\}$ are structures model nodal displacements due to the imposed control-surface deflection. The product $[k_i]\{\Delta_i\}$ in Eq. (15) thus gives the internal forces acting on the ith element due to that imposed deflection. Assuming, as is done in fully stressed design, that internal forces in an element do not change significantly when the element is resized, it may be concluded that the derivative of that element is not significantly dependent upon $[k_i]\{\Delta_i\}$.

The reasoning just presented suggests that the scalar product of the matrices in Eq. (15) changes little when the element is resized, and the derivative, therefore, varies primarily in inverse proportion with the weight of that element. For this reason a value of unity is used for the exponent p in Eq. (3) when resizing for control-surface effectiveness.

Flexible-Surface Lift-Curve Slope

Flexible-surface lift-curve slope may be expressed as the product of the rigid-surface lift-curve slope and a total-surface effectiveness factor. This factor is defined as the ratio of flexible to rigid lift for the entire surface at unit angle of attack. Its value and its derivatives may be computed by using the same relationships just presented for control-surface effectiveness, provided that $\{\alpha\}$ contains unit values everywhere. Thus, the approach used to address the flexible-surface lift-curve-slope parameter becomes a trivial extension to that used for control-surface effectiveness.

Divergence Velocity

If a lifting surface is deformed elastically so as to have an incremental angle-of-attack distribution $\{\Delta\alpha\}$ about some initial equilibrium position, a state of neutral stability will exist if the induced aerodynamic forces are balanced by the induced structural forces. Using previously defined notation, this equilibrium condition requires that

$$q \ulcorner a \lrcorner [AIC]_R \{\Delta\alpha\} = [SIC]^{-1} \{\Delta\alpha\}$$

which may be rewritten in eigenvalue form:

$$\left[[SIC] \ulcorner a \lrcorner [AIC]_R - \lambda[I] \right] \{\Delta\alpha\} = 0 \tag{16}$$

where $\lambda = 1/q$. Solving for the largest, real, positive value of λ gives the critical dynamic pressure q_D, at which static divergence occurs. The corresponding eigenvector $\{\Delta\alpha_D\}$ gives the incremental angle-of-attack distribution at impending divergence.

To facilitate the differentiation of Eq. (16) for the required divergence-velocity derivatives, it is desirable to introduce the adjoint eigenvalue problem of the form

$$\{\beta\}^T \left[[SIC] \ulcorner a \lrcorner [AIC]_R - \lambda[I] \right] = 0 \tag{17}$$

which, it can be shown, has the same eigenvalues as Eq. (16). For the critical eigenvalue, the corresponding eigenvector $\{\beta_D\}^T$ is referred to as the associated row vector of $\{\Delta\alpha_D\}$.

The divergence-velocity derivatives may be obtained by differentiating Eq. (16) with respect to the finite-element weights, premultiplying the resulting expression by $\{\beta\}^T$, and using Eqs. (9) and (17). Furthermore, if the eigenvectors are normalized such that

$$\{\beta_D\}^T\{\Delta\alpha_D\} = 1 \tag{18}$$

the divergence-velocity derivatives may be cast in the following convenient form:

$$\frac{\partial V_D}{\partial w_i} = \frac{q_D}{\rho V_D w_i}\{\bar{\Delta}_{D_i}\}^T[k_i]\{\Delta_{D_i}\} \tag{19}$$

where

$$\{\Delta_D\} = q_D[K]^{-1}[T_F]\lceil a \rfloor [AIC]_R\{\Delta\alpha_D\} \tag{20}$$

and

$$\{\bar{\Delta}_D\} = [K]^{-1}[T_M]\{\beta_D\} \tag{21}$$

In Eq. (19), it has again been assumed that the elements of the stiffness matrix $[K]$ are linear functions of the design variables. Also, the i subscripts in the $\bar{\Delta}_D$ and Δ_D vectors indicate that the only components included are those associated with the degrees of freedom of the ith element.

Examination of the terms appearing in Eq. (20) indicates that $\{\Delta_D\}$ is a column of structures model nodal displacements in the divergence mode. The terms in Eq. (21) appear to represent structures model nodal displacements caused by a set of streamwise moments $\{\beta_D\}$ acting on the aerodynamics model panels. It is difficult to give a physical interpretation to the elements in $\{\beta_D\}$.

The right-hand side of Eq. (19) has the form of a "generalized strain-energy density," similar to that occurring in the expression for the flutter-velocity derivatives in Ref. 1. If the same rationale that is discussed in Ref. 1 is applied to the current derivative expression, it may be reasoned that the divergence-velocity derivative of an element varies, approximately, in inverse proportion with the square of that element's weight. For this reason, a value of ½ is used for the exponent p in Eq. (3), when resizing for divergence velocity.

Applications to Representative Problems

The resizing procedure described has been applied to three problems to demonstrate its ability to address each of the aeroelastic design parameters discussed. The mathematical model of the lifting surface used and the results obtained for it are described in the following paragraphs.

Mathematical Model

The surface selected for study is the wing illustrated in Fig. 1. It is the same structure that has been the subject of previous

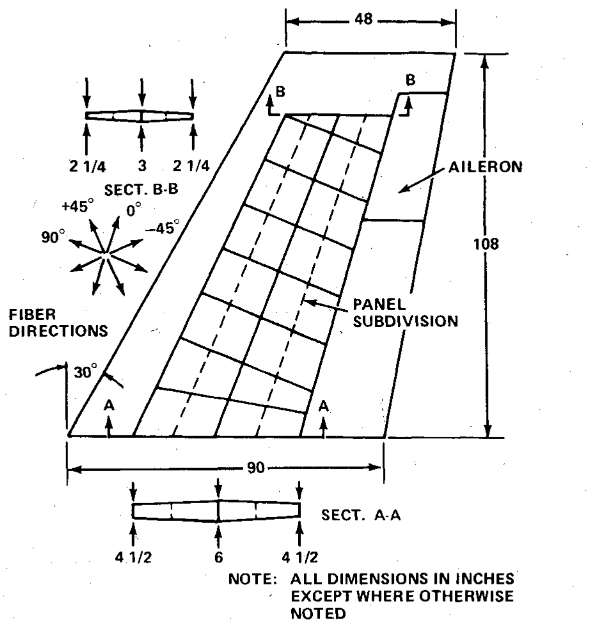

Fig. 1 Aerodynamic planform and primary structural arrangement of wing model.

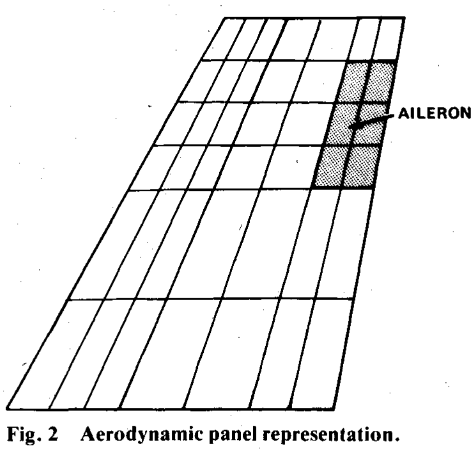

Fig. 2 Aerodynamic panel representation.

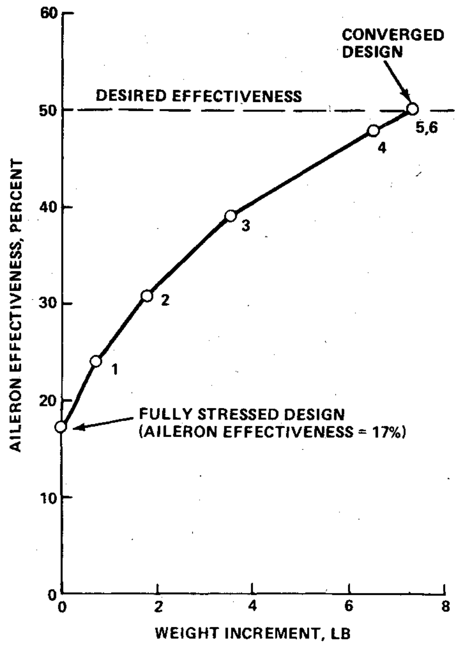

Fig. 3 Summary of resizing steps for improved aileron effectiveness.

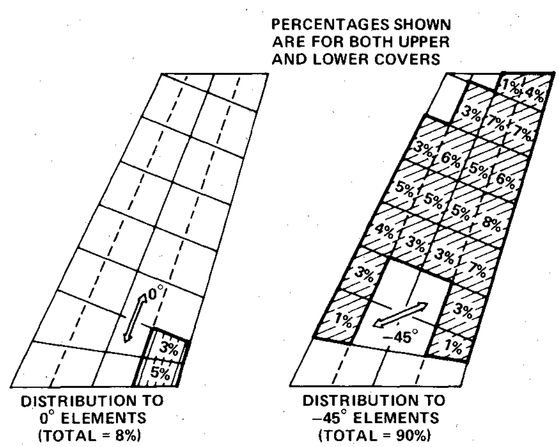

Fig. 4 Distribution of weight increment among elements resized for improved aileron effectiveness.

investigations reported in Refs. 2 and 7. In level of detail, it is representative of what might be used in the earliest stages of preliminary design.

The primary structure consists of a cantilevered, symmetric, two-cell box beam having aluminum substructure and graphite/epoxy cover skins, with fiber directions as shown in Fig. 1. The substructure is modeled with conventional shear panels and posts (which are the only bar elements in the model); the cover skins are modeled with stacked orthotropic membrane elements, each representing the plies in one of the four fiber directions. In all, the model has 234 degrees of freedom and 350 finite elements.

Two applied loading conditions were generated by using simplified pressure distributions representative of a subsonic, forward-center-of-pressure loading and a supersonic, near-uniform-pressure loading. The idealized structure was sized for these conditions by using a fully stressed design computer program. Its resulting weight, exclusive of nonoptimum factors, was 40.9 lb.

With this strength design as a starting point, the static aeroelastic resizing program was then employed to satisfy specific aeroelastic design objectives. The weight of each of the 256 membrane elements in the cover skins was treated as a discrete design variable; only the cover-skin elements were permitted to be resized, and no element thickness was allowed to be reduced below its original strength-based value.

Figure 2 shows the aerodynamics model used in all examples. It contains 42 panels, 6 of which (shown shaded) represent an outboard aileron.

Resizing for Improved Control-Surface Effectiveness

An analysis of the strength-based design indicated that the aeroelastically corrected pressure distribution associated with an imposed aileron deflection produced an aircraft rolling moment that was only 17% of the rigid-wing value (at Mach 0.9, sea level). The automated resizing procedure was then applied to increase the aileron effectiveness to an arbitrary value of 50%. It was desired to achieve this increase in four approximately equal steps; thereafter the procedure was permitted to iterate until the desired effectiveness was satisfied within ±1% for two successive resizing cycles and the weight change between those two cycles was less than 0.1 lb.

Figure 3 summarizes the results of the resizing cycles (obtained in a single computer submission) from the initial fully stressed design to a minimum-weight design having the desired aileron effectiveness. Convergence was obtained in only six steps, with a weight increment of 7.4 lb. As an indication of the relative uniformity achieved among the derivatives of the resized elements, it was observed that the largest and smallest derivatives were 2.4 and 2.1%/lb, respectively.

Figure 4 shows how most of the weight increment was distributed among elements that were resized. Of the total increment, 90% went into the −45-deg (swept aft) elements, primarily in the outer two-thirds of the structure, and 8% went into the 0-deg elements in the trailing-edge root region. This stiffening had the effect of increasing the wing torsional stiffness while also inducing favorable coupling between bending and twisting, thereby reducing the washout that is characteristic of swept-back surfaces.

As a matter of interest, Fig. 5 is provided to show the dramatic influence that the flexibility of the wing has on the pressure distribution for an imposed aileron deflection. The pressures shown correspond to the aerodynamic panels at a wing station through the semispan of the aileron. Flexible-surface pressure distributions are plotted for both the initial fully stressed design (FSD) and the resized design.

In another study, the automated procedure was directed to perform several resizing steps aimed at increasing the aileron effectiveness to a value of 100%, i.e., the rigid-surface value. Figure 6 summarizes the results of seven steps and shows that enormous weight increments are required as the effectiveness increases. A more important point, however, is to be made from this study. Each of the circled points in Fig. 6 corresponds to a design obtained in the course of attempting to raise the effectiveness to 100%. At none of these points was the resizing procedure applied iteratively to enforce convergence to a minimum-weight design at the associated local value of effectiveness. The figure also shows (as points in triangles) minimum-weight converged designs having aileron effectiveness values of 50% (discussed previously) and 60%.

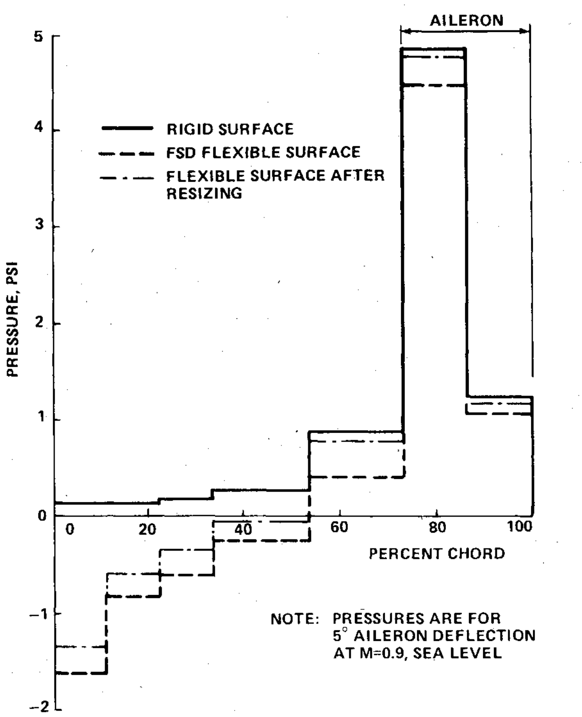

Fig. 5 Chordwise pressure distributions at semispan of aileron.

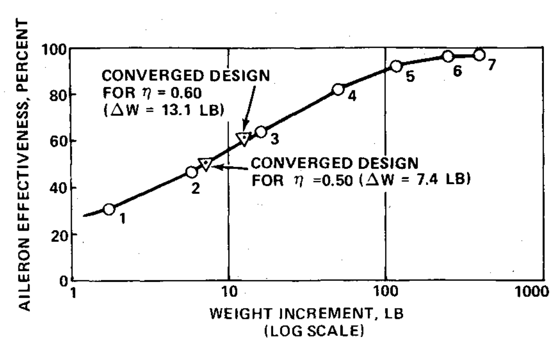

Fig. 6 Redesign history to obtain 100% aileron effectiveness.

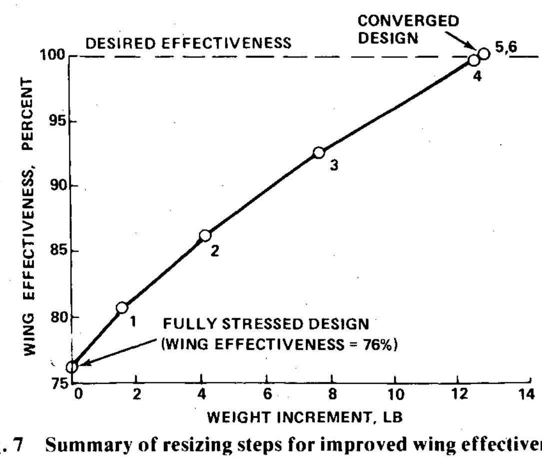

Fig. 7 Summary of resizing steps for improved wing effectiveness.

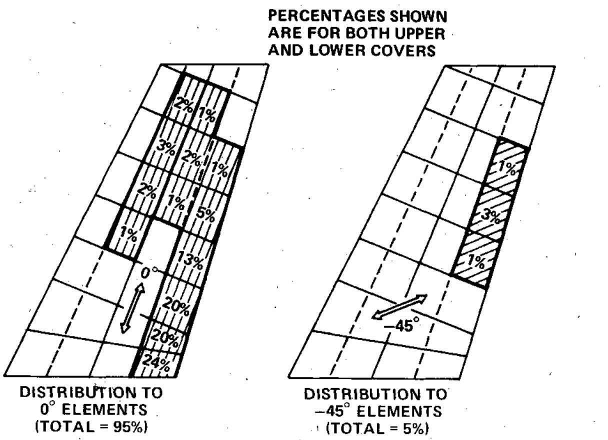

Fig. 8 Distribution of weight increment among elements resized for improved wing effectiveness.

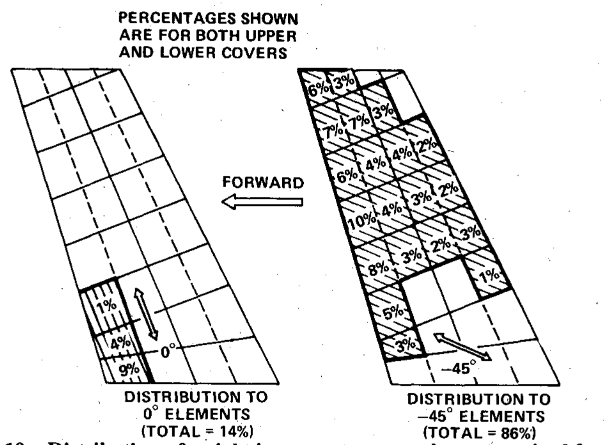

Fig. 10 Distribution of weight increment among elements resized for increased divergence velocity.

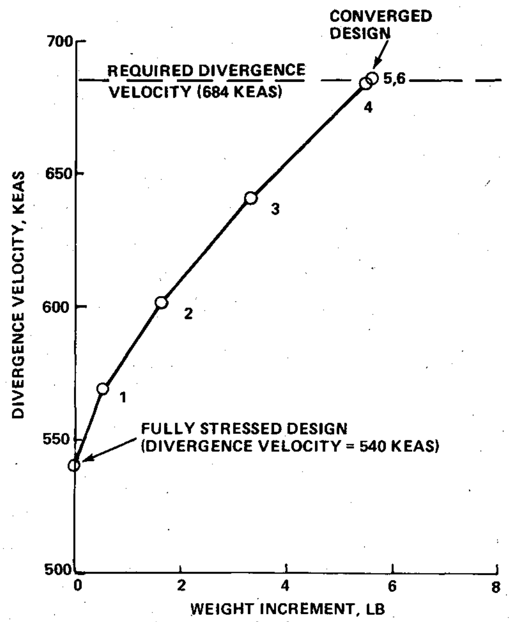

Fig. 9 Summary of resizing steps for increased divergence velocity.

The proximity of these converged points to the curve connecting the circled points indicates that the resizing formula has the inherent property of maintaining near-minimum-weight designs when moving from one value of control-surface effectiveness to another.

Resizing for Improved Total-Surface Effectiveness (Flexible-Surface Lift-Curve Slope)

When the entire wing was placed at angle of attack, the computed flexible-surface lift for the strength design was only 76% of the rigid-surface value (at Mach 1.4, sea level‡). As another demonstration of the use of the automated resizing procedure, it was decided to resize the structure to increase the total-wing effectiveness to its rigid-surface value. The same step size and convergence parameters that were used in the previous example were again used here.

Figure 7 summarizes the results of the resizing steps, with convergence being achieved in only six steps. Figure 8 shows how the weight increment of 12.7 lb was distributed among the resized elements. It may be observed that this distribution is entirely different from the one obtained in the previous example. Here, the major benefit of the stiffening was to reduce bending deformation while simultaneously moving the structural elastic axis aft.

‡It is recognized that this is an unrealistic flight condition; it was selected to provide a suitable combination of pressure distribution and dynamic pressure that would lead to a sizable aeroelastic loss in effectiveness for the starting design.

Resizing for Increased Divergence Velocity

To achieve static divergence of the surface, it was necessary to reverse the direction of flow, creating what was effectively a forward-swept wing. Using the same initial element gages as in the preceding studies, a divergence calculation was then performed for a flight condition at Mach 0.9, sea level. With a 15% safety factor on velocity, this condition requires sea-level flight at 684 knots. However, since the computed divergence speed was only 540 knots, the automated resizing procedure was applied to achieve the required value. As with the previous examples, it was desired to obtain the velocity increase in four approximately equal steps. Convergence was considered to be satisfied when the required divergence speed was met within ± 2 knots and the weight change between two successive resizing cycles was less than 0.1 lb.

A summary of results similar to those presented for the previous examples is given in Figs. 9 and 10. As a matter of additional information, it was observed that the largest and smallest values for the derivatives of the resized elements were 19 and 17 knots/lb, respectively.

Concluding Remarks

The procedure described and demonstrated in this paper offers a practical and highly effective means for determining the lightest way to stiffen a strength-designed structure to meet specific static aeroelastic design objectives. In converting the present pilot computer program into a fully operational design tool, Grumman plans two major improvements.

First, from the point of view of integrated, optimum design, it is desirable to account for interaction between strength and aeroelastic requirements. This interaction will be treated in a way that is similar to that presently used for strength/flutter resizing in the FASTOP system or strength/deflection resizing in the ASOP-3 program. It will then be possible to optimize a design by successively resizing for strength and for the selected aeroelastic constraint, with the strength-design elements from a fully stressed design procedure being considered as minimum gages in the next aeroelastic-constraint resizing, and vice versa.

The second planned improvement concerns the treatment of composite elements. The present approach that requires the program user to define separate orthotropic membrane elements for each ply grouping in a laminate will be replaced by a much more convenient one in which composites are treated explicitly.

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